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STUDY OF INVERSE UNSTEADY STATE THERMO ELASTIC PROBLEM

M. S. Warbhe
Deparment Of Mathematics, Sarvodaya Mahavidyalya, sindewahi Dist.Chandrapur (M.S)

Abstract:
In this paper an attempt is made to determine the temperature, displacement and stress function in a thin disc occupying the space D:{ 0 ≤ r ≤ a , 0 ≤ z ≤ h } by using finite fourier sine transformation Marchi Zagrablich transformation and Laplace transformation.

Introduction:-

Grysa and Kozlowski investigated an inverse one dimensional transient thermo elastic problem and obtained the temperature and heat flux on surface of an isotropic finite slab. Nowacki has determined a stedy state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on upper face with zero temperature on lower face and the circular edge. Roy Choudhary discussed the normal deflection of a thin clamped circular plate due to ramp type heating of concentric circular region of upper face, while lower face of plate is kept at zero temperature and circular edge is thermally insulated. Wankhede has determined the quasi-static thermal stresses in a circular plate subjected to arbitrary temperature on upper face with the lower face at zero temperature and the fixed circular edge thermally insulated.

The investigation in present paper is based on the research papers of which is related to the circular plate is reconstructed for an isotropic annular disc defined as a ≤ r ≤ b , a ≤ z ≤ h and attempt is made to study inverse unsteady state thermo elastic problem to determine the temperature displacement and stress function of disc occupying the space a ≤ r ≤ b , a ≤ z ≤ h by using finite Fourier sine transform Marchi Zagrablich transform and Laplace transformation.

Material Method

Statement Of The Problem : Consider a thin annular disc occupying the space D: { a ≤ r ≤ b , a ≤ z ≤ h }. The initial temperature kept constant which is equal to temperature of the surrounding medium from time t=0 to t=t0. The disc is subjected to a partially distributed and axisymmetric heat supply $\frac{Q_0 f(r,t)}{\Lambda}$ from the interior point (r, z, t)

The differential equation governing the displacement function U(r, z, t) is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+v)a_t T \quad \text{..............................(1)}$$

With $$\frac{\partial U}{\partial r} = 0 \quad \text{at } r = a \text{ and } r = b \quad \text{.........................(2)}$$

Where 'U' and a_t are the Poisson's ratio and linier coefficient of thermal expantion of the material of the plate and T' is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial z} \quad \text{...........................(3)}$$

Subjected to the initial condition

$$\{T(r,z,t)\}_{t=0}=0 \quad \text{...(4)}$$

And the boundary conditions

$$\{T(r,z,t) + k_1\frac{\partial T(r,z,t)}{\partial r}\}_{r=a} = 0 \quad \text{.............................(5)}$$

$$\{T(r,z,t) + k_2\frac{\partial T(r,z,t)}{\partial r}\}_{r=b} = 0 \quad \text{.............................(6)}$$

$$\{T(r,z,t)\}_{z=0} = 0 \quad \text{...(7)}$$

$$\{T(r,z,t)\}_{z=h} = g(r,t) \text{ (unknown)} \quad \text{.......................(8)}$$

The Interior Condition

$$\{T(r,z,t)\}_{z=\xi} = -\frac{Q_0 f(r,t)}{\Lambda} \quad \text{......................................(9)}$$

Where k and Λ are the thermal diffusivity and conductivity of the material of the disc and k_1 and k_2 are radiation constant on the curved surface of the disc respectively. The stress function σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \qquad \text{.....(10)}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \qquad \text{.....(11)}$$

Equation (1) to (11) constitutes the mathematical formulations of thermoelastic problem

Determination of temperature:

Applying Marchi-Zagrablich Transform and Laplace transform and inversion of Laplace Transform and invertion of Marchi-Zagrablich Transform to the equation (3),(4),(7),(8),(9) using equation (5),(6) one obtain the temperature, displacement and stress function.

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left[\frac{2kQ_0}{\Lambda\xi}\left\{\sum_{m=0}^{\infty}\Lambda_m\frac{\sin\Lambda_m z}{\cos\Lambda_m\xi}\right\}\int_0^t \bar{f}(n,t')\, e^{-k(\mu_n^2+\Lambda_m^2)(t-t')}dt'\right] X\ S_0(k_1,k_2,\mu_n r) \quad \text{.....(12)}$$

Similarly we have

$$g(r,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left[\frac{2kQ_0}{\Lambda\xi}\left\{\sum_{m=0}^{\infty}\Lambda_m\frac{\sin\Lambda_m h}{\cos\Lambda_m\xi}\right\}\int_0^t \bar{f}(n,t')\, e^{-k(\mu_n^2+\Lambda_m^2)(t-t')}dt'\right] X\ S_0(k_1,k_2,\mu_n r) \quad \text{........(13)}$$

We have m, n are positive integers

Determination of displacement function:

Substituting the value of *T(r, z, t)* from equation (12) in equation (1) one obtains the thermoelastic displacement function U(r, t, z) as

$$U(r,t,z) = -\frac{2(1+v)kQ_0a_t}{\Lambda\xi}\sum_{n=1}^{\infty}\frac{1}{C_n\mu_n^2}\left\{\sum_{l=0}^{\infty}\Lambda_l\frac{\sin\Lambda_l z}{\cos\Lambda_l\xi}\right\}\left[\int_0^t \bar{f}(n,t')\, e^{-k(\mu_n^2+\Lambda_l^2)(t-t')}dt'\right] X\ (k_1,k_2,\mu_n r) \text{........(14)}$$

Determination of stress function:

Substituting the value of (14) in (10), (11) one obtains the stress function as,

$$\sigma_{rr} = \frac{4\mu(1+v)kQ_0a_t}{\Lambda\xi}\sum_{n=1}^{\infty}\frac{1}{C_n\mu_n^2}\left\{\sum_{l=0}^{\infty}\Lambda_l\frac{\sin\Lambda_l z}{\cos\Lambda_l\xi}\right\}\left[\int_0^t \bar{f}(n,t')\, e^{-k(\mu_n^2+\Lambda_l^2)(t-t')}dt'\right] X\ \frac{1}{r}S_0'(k_1,k_2,\mu_n r) \text{......(15)}$$

$$\sigma_{\theta\theta} = \frac{4\mu(1+v)kQ_0a_t}{\Lambda\xi}\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{l=0}^{\infty}\Lambda_l\frac{\sin\Lambda_l z}{\cos\Lambda_l\xi}\right\}\left[\int_0^t \bar{f}(n,t')\, e^{-k(\mu_n^2+\Lambda_l^2)(t-t')}dt'\right] X\ S_0'(k_1,k_2,\mu_n r) \text{.........(16)}$$

We have m, n are positive integers

Conclusion:

By using Machi- Zagrablich Transform and Laplace transform technique temperature, displacement and stress functions can be obtained at any point of the disc, when interior temperature and other boundary conditions are known.

The result presented here will be more useful in engineering problems particularly in the determination of state of strain in the disc constituting the foundations of container for hot gases or liquid in foundations for furnaces etc.

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References:

1. K. C. Deshmukh and S.A.Meshram: Inverse thermostatic problem in a thin circular plate. Bull. of Pure and Appl. Sci. Vol. 20E(No.1),(2001), pp, 113-122.
2. Grysa K and Kozlowski Z: One dimensional problem of temperature and heat flux determination at surface of thermoelastic Slab part-I, The analytical solutions, NUCL, Engrg 74(1982), pp. 1-14
3. Nowacki W: The state of stress in a thick circular plate due to temperature field. Bull. Sci. Acad. Polon. Ser. Tech. 5, (1957), p227
4. Patel S.R.: Inverse problems of transient heat conduction with radiation, the Mathematics Education Vol.5, No.4,(1971),pp. 85-90.
5. E. Marchi and G.Zagrablich, Heat conduction in hollow cylinders with radiation, Proc.Sindurdurgh Math. Soc. 14(2) (1964), 159-164
6. Roy Choudhuri S. K.: A note on a quasi-static stresses in a thin circular plate due to transient temperature applied along the circumference of a circle over upper face. Bull. Acad. Sci. Ser. Tech. 20(1972) pp.21-24.
7. Wankhede P.C.: ON the quasi-static thermal stresses in a circular plate. Indian J. of Pure and Appl. Maths 13, (1982) pp. 1273-1277.